

Hybrid quantum device with nitrogen-vacancy centers in diamond coupled to carbon nanotubes

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We show that NV centers in diamond interfaced with a suspended carbon nanotube carrying a dc current can facilitate a spin-nanomechanical hybrid device. We demonstrate that strong magnetomechanical interactions between a single NV spin and the vibrational mode of the suspended nanotube can be engineered and dynamically tuned by external control over the system parameters. This spin-nanomechanical setup with strong, *intrinsic* and *tunable* magnetomechanical couplings allows for the construction of hybrid quantum devices with NV centers and carbon-based nanostructures, as well as phonon-mediated quantum information processing with spin qubits.

Carbon-based structures and devices are very commonly used in our everyday life and in state-of-the-art science and technology. In quantum information science, nitrogen-vacancy (NV) centers in diamond are outstanding solid state qubits due to their long coherence times and high controllability [1–5]. In nano-mechanics, mechanical resonators made out of allotropes of carbon (such as nanotubes [6–8], diamond [9–11], and graphene [12]), are being extensively studied for fundamental research and practical applications [13–23].

Recently, much attention has been paid to coupling NV spins in diamond to mechanical resonators, which can be achieved extrinsically [24–34] or intrinsically [35–39]. In the first case, the interaction arises from the relative motion of the NV spin and a source of local magnetic field gradients [24]. In such setups, a magnetic tip mounted on a vibrating cantilever [40] is often used to generate the magnetic coupling between an NV spin and the mechanical motion [24–34]. However, creating very strong, well-controlled, local gradients remains challenging for such setups, in particular when arrays of NV centers are placed in close proximity to the same cantilever. Thus far, experiments with the extrinsic coupling scheme have yet to reach the strong-coupling regime [26–29]. In the second case, the coupling of a diamond cantilever to the spin of an embedded NV center is induced by crystal strain during mechanical motion [35–39]. Unfortunately, the strain-induced interaction between a single NV spin and the cantilever quantized motion is inherently tiny [37, 38], which makes the strong strain coupling at a single quantum level very challenging.

In this Letter, we propose that NV centers in diamond interfaced with carbon nanotubes can facilitate a spin-nanomechanical hybrid device. This hybrid structure takes advantage of the unprecedented mechanical and electrical characteristics of carbon nanotubes, as well as the exceptional coherence properties of NV centers in diamond. We demonstrate that the physics of an NV center in diamond placed near a carbon nanotube

with a dc current flowing through it can be well mapped to cavity quantum-electrodynamics (QED). In particular, going beyond earlier work in this field [24–39], we show that the magnetomechanical interaction can be engineered and dynamically tuned by external control of the driving microwave fields and electric current through the nanotube. The resulting coupling strength can be roughly three orders of magnitude stronger than that for cold atoms coupled to nanowires [16, 17]. An inherent advantage of our NV-nanotube hybrid system is the intrinsic nature of the coupling. Thus no additional components, such as external magnetic tips, are required to tune the coupling. Another distinct feature of this intrinsic coupling scheme is that it is scalable to arrays of NV centers in diamond. This spin-nanomechanical structure with strong intrinsic magnetomechanical couplings would open up new avenues towards the design of hybrid quantum devices [41] with NV centers and carbon-based nanostructures. It also allows for quantum information processing with NV spin qubits [42, 43], and could serve as novel nanoscale sensors [44–48] in physical and life science.

Model.— We consider a setup as shown in Fig. 1(a), where the magnetic field of a current-carrying nanotube is coupled to an NV center spin embedded in a nanodiamond. The nanotube of length L is suspended along the x axis at a distance d from the diamond nanocrystal. When it vibrates, d varies by the nanotube's effective

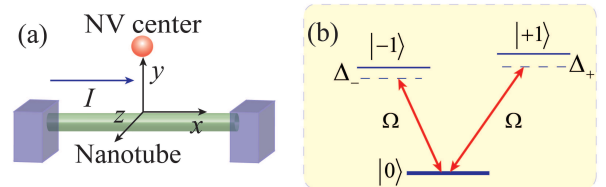


FIG. 1. (Color online) (a) Schematic of a single NV center in a diamond nanocrystal located near a current-carrying nanotube. (b) Level diagram of the driven NV center.

transverse displacement. In the following, we assume the transverse displacement to be along the y direction, and express it with the oscillator operator \hat{a} of the fundamental oscillating mode, i.e., $\hat{u}_y = (\hbar/2m\omega_{\text{nt}})^{1/2}(\hat{a} + \hat{a}^\dagger)$, where m is the effective mass of the nanotube, and ω_{nt} is the mechanical vibration frequency [49]. The magnetic field $\vec{B}_{\text{nt}}(\vec{r})$ of the current-carrying nanotube at position \vec{r} can be calculated by the Biot-Savart law. For a long nanotube ($L \gg d$), the magnetic field has the form $\vec{B}_{\text{nt}}(\vec{r}) = \mu_0 I \vec{e}_x \times \vec{r} / 2\pi |\vec{r}|^2$, in a reference frame with axes as in Fig. 1. Here \vec{e}_x is the unit vector in the x direction, and I is the electric current in the nanotube.

NV centers in diamond consist of a substitutional nitrogen atom and an adjacent vacancy, which have a spin $S = 1$ ground state, with zero-field splitting $D = 2\pi \times 2.87$ GHz, between the $|m_s = \pm 1\rangle$ and $|m_s = 0\rangle$ states. For moderate applied magnetic fields, static and low frequency components of magnetic fields B_z cause Zeeman shifts of states $|m_s = \pm 1\rangle$, while external microwave fields with magnetic field \vec{B}_{dr} drive Rabi oscillations between $|m_s = 0\rangle$ and the excited states $|m_s = \pm 1\rangle$, as shown in Fig. 1(b). For convenience, we denote as the z axis the crystalline axis of the NV center.

The interaction of a single NV center located at \vec{r} with the total magnetic field (external driving and from the nanotube) can be written as $\hat{H}_{\text{NV}} = \hbar D \hat{S}_z^2 + \mu_B g_s B_z \hat{S}_z + \mu_B g_s (\vec{B}_{\text{nt}}(\vec{r}) + \vec{B}_{\text{dr}}) \cdot \hat{\vec{S}}$ with $g_s = 2$ the Landé factor of the NV center, μ_B the Bohr magneton, and $\hat{\vec{S}}$ the spin operator of the NV center. We consider using a single microwave field polarized in the x direction, $\vec{B}_{\text{dr}} = B_0 \cos \omega_0 t \vec{e}_x$, and place the NV center in the position where the magnetic field of the nanotube is in the z direction, $\vec{B}_{\text{nt}} = B_{\text{nt}} \vec{e}_z$. The magnetic field $\vec{B}_{\text{nt}}(\vec{r})$ felt by the NV center is modulated by the nanotube's vibration. Expanding the magnetic field $\vec{B}_{\text{nt}}(\vec{r})$ up to first order in \hat{u}_y , we have $\hat{H}_{\text{NV}} = \hbar D \hat{S}_z^2 + \mu_B g_s [B_z + B_{\text{nt}}(d)] \hat{S}_z + \mu_B g_s \vec{B}_{\text{dr}} \cdot \hat{\vec{S}} + \mu_B g_s \hat{S}_z \partial_y B_{\text{nt}} \hat{u}_y$.

We define $\hbar \Delta_{\pm} = \hbar D \pm \mu_B g_s (B_z + B_{\text{nt}}) - \hbar \omega_0$, $\hbar \Omega = \frac{\sqrt{2}}{4} \mu_B g_s B_0$, and restrict the following discussion to symmetric detunings, $\Delta_+ = \Delta_- = \Delta$. When $|\Delta| \gg \Omega$, we obtain the effective Hamiltonian [49]

$$\hat{\mathcal{H}}_q = \hbar \omega_{\text{nt}} \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \Lambda \hat{\sigma}_z + \hbar g (\hat{\sigma}_+ + \hat{\sigma}_-) (\hat{a}^\dagger + \hat{a}). \quad (1)$$

Here, $\Lambda = 2\Omega^2/\Delta$, $\hbar g = \mu_B g_s (\hbar/2m\omega_{\text{nt}})^{1/2} \partial_y B_{\text{nt}}$, and we switch to the new basis $\{|B\rangle = \frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle), |D\rangle = \frac{1}{\sqrt{2}}(|+1\rangle - |-1\rangle)\}$, with $\hat{\sigma}_z = |B\rangle\langle B| - |D\rangle\langle D|$, $\hat{\sigma}_+ = |B\rangle\langle D|$, and $\hat{\sigma}_- = |D\rangle\langle B|$. The states $|B\rangle$ and $|D\rangle$ are often referred to bright and dark states for NV spins [24, 57]. If we choose $\Lambda \simeq \omega_{\text{nt}}$, then, under the rotating-wave approximation we obtain the standard Jaynes-Cummings (JC) Hamiltonian

$$\hat{\mathcal{H}}_{\text{JC}} = \hbar \omega_{\text{nt}} \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \Lambda \hat{\sigma}_z + \hbar g (\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger). \quad (2)$$

However, if we choose $\Lambda \simeq -\omega_{\text{nt}}$, which can be controlled by the parameters Δ and Ω , we obtain the Anti-Jaynes-Cummings (AJC) Hamiltonian

$$\hat{\mathcal{H}}_{\text{AJC}} = \hbar \omega_{\text{nt}} \hat{a}^\dagger \hat{a} + \frac{1}{2} \hbar \Lambda \hat{\sigma}_z + \hbar g (\hat{\sigma}_+ \hat{a}^\dagger + \hat{\sigma}_- \hat{a}). \quad (3)$$

Thus, our system mimics the standard model in cavity QED of a single atom coupled to a single cavity mode. The type of interactions can be designed by external control over the driving fields. This mapping allows the powerful toolbox of cavity QED to be transferred to these systems.

Two proposed experimental setups.— We consider two different designs for coupling an NV spin to a current-carrying carbon nanotube resonator. Fig. 2 (a) displays a nanotube, carrying a dc current, suspended above a bulk single crystal diamond sample. Individual, optically-resolvable NV centers are implanted 5–10 nm below the surface of the diamond sample [27, 58]. Fig. 2 (b) shows another feasible design, where a diamond nanoparticle hosting a single NV center is closely placed near the nanotube. Diamond nanoparticles can have a size of less than 10 nanometers, and only host one NV defect [26]. The spin states of NV centers can be controlled and manipulated by microwaves from external microwave antennas. A confocal microscope can be used to excite and polarize the NV spin, and detect photoluminescence to read out the NV spin polarization.

Carbon nanotubes can possess current-carrying capacities exceeding $10 \mu\text{A}/\text{nm}^2$ [59–64], with lengths which can range from tens of nanometers to tens of micrometers. In experiments, the carbon nanotube could be actuated and deflected electrostatically over several nanometers with AC and DC voltages applied to the gate electrodes. Thus, the distance between the nanotube and the NV center can be fine-tuned electrostatically. A recent experiment [23], has reported a device with a graphene membrane suspended some 10–50 nm above a single NV center.

To evaluate the single spin-phonon coupling strength g , we use the magnetic field generated by an infinite long tube, as given by the Biot-Savart law. In this case, it

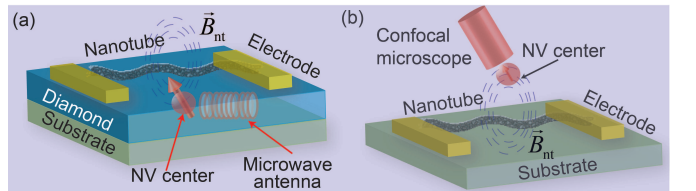


FIG. 2. (Color online) Schematic of the nanotube-NV hybrid setup. (a) A current-carrying nanotube is suspended above a diamond sample, in which individual optically-resolvable NV centers are implanted 5–10 nm below its surface. (b) A single NV defect hosted in a diamond nanocrystal with a size about 10 nm is positioned near the nanotube.

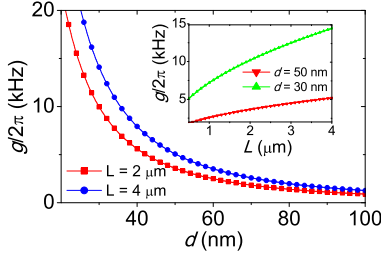


FIG. 3. (Color online) Single spin-phonon coupling strength versus the distance between the NV center and the nanotube. The relevant parameters here are $r \sim 1.5$ nm, $t \sim 0.335$ nm, $I \sim 60$ μ A. In the inset the coupling strength versus the length of the nanotube.

reads

$$g = \frac{\mu_B g_s \mu_0 I}{2\pi \sqrt{2\hbar m \omega_{\text{nt}}} d^2}. \quad (4)$$

This magnetomechanical coupling strength depends on the dimensions of the nanotube and the distance d , as well as the current I flowing through the nanotube. Thus, it can be easily tuned by control of the system parameters.

We consider a carbon nanotube of length $L \sim 2$ μ m, radius $r \sim 1.5$ nm, and wall thickness $t \sim 0.335$ nm, suspended at a distance $d \sim 30$ nm from the NV center [49]. The tube carries a dc current $I \sim 60$ μ A, and vibrates at a frequency $\omega_{\text{nt}}/2\pi \sim 2$ MHz, with effective mass $m \sim 7 \times 10^{-21}$ kg [49]. With the above given parameters, one can obtain $g/2\pi \sim 10$ kHz. By changing the distance d and the dimensions of the nanotube, as well as the current I flowing through the nanotube, the coupling strength g can be further adjusted (see Fig. 3). For a much closer distance $d \sim 10$ nm, the magnetomechanical coupling strength can even reach $g/2\pi \sim 100$ kHz. This coupling strength is comparable to that of a single NV spin coupled to a vibrating cantilever with a strong local magnet [24, 25] or a superconducting circuit [65–70], and is about a factor of 1000 larger than the coupling achieved with cold atoms [16, 17].

Dephasing and dissipation.— In realistic situations, we need to consider spin dephasing and mechanical dissipation. The full dynamics of our system that takes these incoherent processes into account is described by the master equation

$$\begin{aligned} \frac{d\hat{\rho}(t)}{dt} = & -\frac{i}{\hbar} [\hat{H}_{\text{JC}}, \hat{\rho}] + \gamma_s \mathcal{D}[\hat{\sigma}_z] \hat{\rho} \\ & + n_{\text{th}} \gamma_m \mathcal{D}[\hat{a}^\dagger] \hat{\rho} + (n_{\text{th}} + 1) \gamma_m \mathcal{D}[\hat{a}] \hat{\rho} \end{aligned} \quad (5)$$

with $\mathcal{D}[\hat{o}] \hat{\rho} = \hat{o} \hat{\rho} \hat{o}^\dagger - \frac{1}{2} \hat{o}^\dagger \hat{o} \hat{\rho} - \frac{1}{2} \hat{\rho} \hat{o}^\dagger \hat{o}$ for a given operator \hat{o} . The strong coupling regime can be reached if the coherent coupling strength g exceeds both the electronic spin decay rate γ_s and the intrinsic damping rate of the mechanical mode γ_m , i.e., $g > \{\gamma_s, n_{\text{th}} \gamma_m\}$, with

$n_{\text{th}} = (e^{\hbar \omega_{\text{nt}}/k_B T} - 1)^{-1}$ the thermal phonon number at the environment temperature T . For a mechanical resonator with frequency ω_m and quality factor Q , the mechanical damping rate is $\gamma_m = \omega_m/Q$. The recent fabrication of carbon nanotube resonators can possess quality factors exceeding 10^5 [8]. Together with the oscillator frequency $\omega_{\text{nt}}/2\pi \sim 2$ MHz [49], this value of Q implies an oscillator damping rate $\gamma_m/2\pi \sim 20$ Hz, and would translate into phonon mean free path $l_c \sim QL \sim 10$ cm [49]. Assuming an environmental temperature $T \sim 10$ mK in a dilution refrigerator, the thermal phonon number is about $n_{\text{th}} \sim 100$. Therefore, we obtain $g > n_{\text{th}} \gamma_m$. When it comes to the NV center, the dephasing time T_2 can be increased to several milliseconds in ultrapure diamond [71], leading to a dephasing rate $\gamma_s/2\pi \sim 1$ kHz. We can ignore single spin relaxation, as T_1 can be several minutes at low temperatures. Therefore, the strong-coupling regime can be reached in this setup, i.e., $g > \{\gamma_s, n_{\text{th}} \gamma_m\}$.

The strong-coupling regime described by the Hamiltonian (2) enables coherent quantum state transfer between the spin and the resonator. Moreover, in combination with optical pumping and detection techniques for spin qubits, this would provide the basic ingredients for detecting and manipulating quantum states of the nanotube resonator.

In Fig. 4, we show the numerical simulations of quantum dynamics of the coupled system through solving the master equation (5). As the initial state, we take the product state of the NV spin and the mechanical resonator with the occupation number $n_m = 0.2$, e.g., as a result of side-band cooling [72, 73]. In the time domain, vacuum Rabi oscillations are a direct evidence of the coherent energy exchange between the spin qubit and the resonator phonon mode. We obtain numerical results for the time evolution of the mean phonon number and the probability for the NV center being in the excited state. We find that vacuum Rabi oscillations can occur for these parameters. In such a process, the spin state

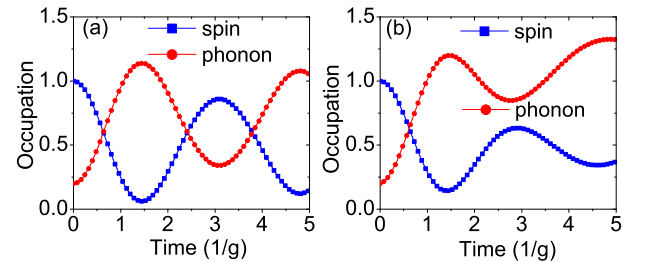


FIG. 4. (Color online) (a) Vacuum Rabi oscillations of an NV spin coupled to a nanotube mechanical resonator without dissipations. The initial state of the NV spin is $|\mathcal{B}\rangle$. (b) Same as (a) but with dissipations for the spin and the mechanical resonator. The relevant parameters here are chosen as $n_{\text{th}} \sim 100$, $\gamma_m \sim 10^{-3}g$, and $\gamma_s \sim 0.1g$.

can be transferred from the NV center to the nanotube resonator, and vice versa.

Elementary quantum information.— The magnetomechanical interaction allows us to use the nanotube vibration mode as a quantum bus to perform more complex tasks. Controlled spin-spin couplings could be realized for two distant NV centers separated by micrometer distances. Based on these effective interactions, we now explore the possibility of implementing quantum information processing with spin qubits.

We consider two separated NV centers coupled to the same vibration mode of the nanotube in the dispersive regime $|\Lambda - \omega_{\text{nt}}| \gg g$. This will lead to an effective long range spin-spin interaction via the exchange of virtual phonons [49], $\hat{\mathcal{H}}_{\text{s-s}} = \hbar\lambda_{\text{eff}}(\hat{\sigma}_+^1\hat{\sigma}_-^2 + \hat{\sigma}_-^1\hat{\sigma}_+^2)$, with the coupling strength $\lambda_{\text{eff}} = g^2/|\Lambda - \omega_{\text{nt}}|$. The coherence length of the phonon mediated NV spin coupling is about $l_c \sim QL$ [49], which can be much larger than the distance between two NV spins separated by a distance on the order of the nanotube's length. If we choose $|\Lambda - \omega_{\text{nt}}|/2\pi \sim 1$ MHz, and $g/2\pi \sim 100$ kHz, then we can obtain the spin-spin coupling strength $\lambda_{\text{eff}}/2\pi \sim 10$ kHz. This strong spin-spin interaction allows for the implementation of a SWAP gate and quantum states transfer between two NV centers.

In the following, we encode the j th logical qubit in the two spin states of the j th NV center, i.e., $|0\rangle_q^j = |0\rangle_j$ and $|1\rangle_q^j = |\mathcal{D}\rangle_j$. Such qubit encoding has proven to be more robust against low-frequency magnetic-field noise [57]. Our main task is to realize a SWAP gate and quantum information transfer between two qubits. To implement this protocol, we need a microwave to drive the transition between the qubit state $|0\rangle_j$ and the bright state $|\mathcal{B}\rangle_j$ in each qubit with Rabi frequency Ω_j and frequency detuning δ_j . The dynamics of the entire system is described by $\hat{\mathcal{H}} = \sum_j \hbar\delta_j|\mathcal{B}\rangle_{jj}\langle\mathcal{B}| + \sum_j [\hbar\Omega_j|\mathcal{B}\rangle_{jj}\langle 0| + \text{H.c.}] + \hat{\mathcal{H}}_{\text{s-s}}$. The spin-spin interaction can be diagonalized with the states $|\pm\rangle_q = 1/\sqrt{2}[|\mathcal{B}\rangle_1|\mathcal{D}\rangle_2 \pm |\mathcal{D}\rangle_1|\mathcal{B}\rangle_2]$. It is easy to show that in the subspace defined by $\{|0,1\rangle_q \equiv |0\rangle_q^1|1\rangle_q^2, |+\rangle_q, |-\rangle_q, |1,0\rangle_q \equiv |1\rangle_q^1|0\rangle_q^2\}$, the system Hamiltonian has the form [49]

$$\begin{aligned} \hat{\mathcal{H}} = & \hbar\delta_+|+\rangle_{qq}\langle +| - \hbar\delta_-|-\rangle_{qq}\langle -| + \hbar\bar{\Omega}_1|+\rangle_{qq}\langle 0,1| \\ & + \hbar\bar{\Omega}_1|-\rangle_{qq}\langle 0,1| + \hbar\bar{\Omega}_2|+\rangle_{qq}\langle 1,0| \\ & - \hbar\bar{\Omega}_2|-\rangle_{qq}\langle 1,0| + \text{H.c.} \end{aligned} \quad (6)$$

with $\delta_+ = \lambda_{\text{eff}} + \frac{\delta_1 + \delta_2}{2}$, $\delta_- = \lambda_{\text{eff}} - \frac{\delta_1 + \delta_2}{2}$, $\bar{\Omega}_j = \Omega_j/\sqrt{2}$, $j = 1, 2$. Thus we can find that if the two qubits are initially prepared in the state $|0\rangle_q^1|1\rangle_q^2$ or $|1\rangle_q^1|0\rangle_q^2$, then the dynamics of the system will be confined in the subspace governed by the Hamiltonian (6). In Fig. 5 we present detailed numerical simulations for the dynamics of the coupled system. It can be found that at the moment $T_{\text{sw}} = \pi/\lambda_{\text{eff}}$, the system evolves from the state $|0\rangle_q^1|1\rangle_q^2$ to the state $|1\rangle_q^1|0\rangle_q^2$ via the intermediate states $|\pm\rangle_q$ through microwave driving, and vice versa.

The state $|1\rangle_q^1|1\rangle_q^2$ remains unchanged during this process, while the system will be brought from the state $|0\rangle_q^1|0\rangle_q^2$ to the state $|\mathcal{B}\rangle_1|0\rangle_q^2$ or $|0\rangle_q^1|\mathcal{B}\rangle_2$, and back again at the moment T_{sw} . Therefore, in the language of quantum information theory, this operation corresponds to a SWAP gate. This gate can be exploited to realize quantum state transfer between two NV spins, i.e., $(\alpha|0\rangle_q^1 + \beta|1\rangle_q^1)|1\rangle_q^2 \rightarrow |1\rangle_q^1(\alpha|0\rangle_q^2 + \beta|1\rangle_q^2)$.

The gate fidelity is limited by the following factors: (i) spin decoherence induced by phonon excitations with an effective decay rate $\Gamma \simeq n_{\text{th}}\gamma_m g^2/|\Lambda - \omega_{\text{nt}}|^2$; (ii) single spin dephasing due to low-frequency noise with a dephasing rate γ_s , which is assumed to be Markovian for simplicity. Recent work has shown that by coupling a single NV spin to another two-level system and encoding quantum information in the dark-states $|0\rangle$ and $|\mathcal{D}\rangle$, the coherent time T_2 can be prolonged [57]. For isotopically purified diamond, we can safely choose $T_2 \sim 1$ ms. Taking these factors together, we find the gate fidelity can be estimated as $F \sim (1 - T_{\text{sw}}\Gamma - T_{\text{sw}}/T_2) > 0.95$ for the given parameters, with an operating time $T_{\text{sw}} \sim 50 \mu\text{s}$.

Conclusions.— We have proposed a spin-nanomechanical hybrid device where a single NV center spin in diamond is coupled to the vibrational mode of a suspended current-carrying carbon nanotube. It makes the strong spin-mechanical coupling at a single quantum level feasible, and allows fast mechanical control of spin qubits and efficient phonon cooling by an NV center. Such a device can find applications in phonon-mediated quantum information processing with NV spin qubits, and could serve as novel nanoscale sensors for detecting tiny pressure, temperature, electric and magnetic field changes.

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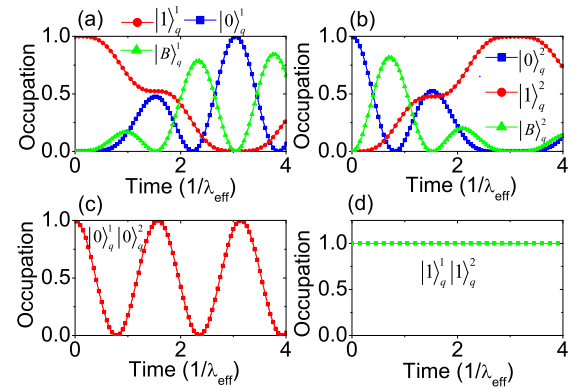


FIG. 5. (Color online) (a) Time evolution of the probabilities for the first NV center being in the states $|1\rangle$, $|0\rangle$, and $|\mathcal{B}\rangle$. (b) Same as in (a) but for the second NV center. In both cases, the initial state is chosen as $|1\rangle_q^1|0\rangle_q^2$. (c) and (d): Time evolution of the probabilities for the two NV spins in the states $|0\rangle_q^1|0\rangle_q^2$, and $|1\rangle_q^1|1\rangle_q^2$ respectively. The relevant parameters here are $\Omega_1 \simeq \Omega_2 = 2\lambda_{\text{eff}}$, and $\delta_1 = \delta_2 = 0$.

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SUPPLEMENTAL MATERIAL:

Fundamental vibration mode of the carbon nanotube

The motion of a suspended nanotube can be described by the Euler-Bernoulli theory [S1, S2]. The Euler-Bernoulli equation for the static and dynamic displacement of a thin beam subjected to an external driving force reads

$$\rho A \frac{\partial^2}{\partial t^2} \phi(x, t) + E\mathcal{I} \frac{\partial^4}{\partial x^4} \phi(x, t) - T \frac{\partial^2}{\partial x^2} \phi(x, t) = F_{\text{ext}}(x, t), \quad (\text{S1})$$

where $\phi(x, t)$ is the lateral displacement in the y direction, ρ the mass density, A the beam cross section, E the Young modulus, \mathcal{I} the moment of inertia, T the tension in the tube, and $F_{\text{ext}}(x, t)$ a unit length force that accounts for the effect of the gate electrodes. The displacement amplitude increases remarkably when the tube is driven at an eigenfrequency of the system. These eigenfrequencies correspond to the bending modes of the nanotube, which we label by mode number n starting from zero for the fundamental mode. We here consider the eigenfrequencies of a perfect clamping nanotube resonator, in which case the tension T goes to zero at zero gate voltage $F_{\text{ext}}(x, t) = 0$ [S3, S4]. In this case the Euler-Bernoulli equation reads

$$\rho A \frac{\partial^2}{\partial t^2} \phi(x, t) + E\mathcal{I} \frac{\partial^4}{\partial x^4} \phi(x, t) = 0. \quad (\text{S2})$$

The eigenmodes ϕ_n and eigenfrequencies ω_n satisfy:

$$\omega_n^2 \rho A \phi_n = E\mathcal{I} \frac{\partial^4}{\partial x^4} \phi_n \quad (\text{S3})$$

The solutions to this equation are

$$\phi_n(x) = C_1(\cos k_n x - \cosh k_n x) + C_2(\sin k_n x - \sinh k_n x). \quad (\text{S4})$$

For a doubly clamped beam, the boundary conditions are $\phi_n(0) = \phi_n(L) = 0, \phi'_n(0) = \phi'_n(L) = 0$, and for a cantilever the boundary conditions are $\phi_n(0) = \phi'_n(0) = 0, \phi''_n(L) = \phi'''_n(L) = 0$. For the former case, the frequency equation is given by

$$\cos k_n L \cosh k_n L = 1. \quad (\text{S5})$$

For the latter, the frequency equation is given by

$$\cos k_n L \cosh k_n L = -1. \quad (\text{S6})$$

The first five nontrivial consecutive roots of these equations are given below

Mode	Cantilever	Beam
n	$k_n L$	$k_n L$
0	1.875	4.730
1	4.694	7.853
2	7.855	10.996
3	10.996	14.137
4	14.137	17.279

The corresponding eigenfrequencies are

$$\omega_n = k_n^2 \sqrt{\frac{E\mathcal{I}}{\rho A}}. \quad (\text{S7})$$

Therefore, the fundamental vibrational mode of a nanotube has the vibration frequency $\omega_{\text{nt}} \sim \frac{1}{L^2} \sqrt{\frac{E\mathcal{I}}{\rho A}}$ [S3, S4]. In table I we present the relevant parameters for the carbon nanotube resonator without gate voltages.

TABLE I. Parameters for the nanotube considered in this work.

Term	Value	Units
Length L	2	μm
Radius r	1.5	nm
Wall thickness t	0.335	nm
Mass density ρ	1350	kg/m^3
Effective mass m	7×10^{-21}	kg
Young modulus E	1	TPa
Fundamental frequency ω_0	$2\pi \times 2$	MHz
Current carrying capacity C	≥ 10	$\mu\text{A/nm}^2$
Current I	60	μA
Quality factor Q	10^5	/

Derivation of the spin-vibration interaction

The interaction of a single NV center located at \vec{r} with the total magnetic field (external driving and from the nanotube) is

$$\hat{H}_{\text{NV}} = \hbar D \hat{S}_z^2 + \mu_B g_s B_z \hat{S}_z + \mu_B g_s (\vec{B}_{\text{nt}}(\vec{r}) + \vec{B}_{\text{dr}}) \cdot \hat{\vec{S}}. \quad (\text{S8})$$

Expanding the magnetic field $\vec{B}_{\text{nt}}(\vec{r})$ up to first order in \hat{u}_y , we have

$$\hat{H}_{\text{NV}} = \hbar D \hat{S}_z^2 + \mu_B g_s [B_z + B_{\text{nt}}(d)] \hat{S}_z + \mu_B g_s \vec{B}_{\text{dr}} \cdot \hat{\vec{S}} + \mu_B g_s \hat{S}_z \partial_y B_{\text{nt}} \hat{u}_y. \quad (\text{S9})$$

In the basis defined by the eigenstates of \hat{S}_z , i.e., $\{|m_s\rangle, m_s = 0, \pm 1\}$, with $\hat{S}_z|m_s\rangle = m_s|m_s\rangle$, we get

$$\begin{aligned} \hat{H}_{\text{NV}} = & \sum_{m_s} \{ \langle m_s | [\hbar D \hat{S}_z^2 + \mu_B g_s [B_z + B_{\text{nt}}(d)] \hat{S}_z] | m_s \rangle \} |m_s\rangle \langle m_s| \\ & + \sum_{m_s, m'_s} \{ \langle m_s | \mu_B g_s \vec{B}_{\text{dr}} \cdot \hat{\vec{S}} | m'_s \rangle \} |m_s\rangle \langle m'_s| + \sum_{m_s} \{ \langle m_s | \mu_B g_s \hat{S}_z \partial_y B_{\text{nt}} \hat{u}_y | m_s \rangle \} |m_s\rangle \langle m_s|. \end{aligned} \quad (\text{S10})$$

Taking $\vec{B}_{\text{dr}} = B_0 \cos \omega_0 t \vec{e}_x = B_0/2 (e^{i\omega_0 t} + e^{-i\omega_0 t}) \vec{e}_x$, we have

$$\begin{aligned} \hat{H}_{\text{NV}} = & \sum_{m_s} \{ \hbar D m_s^2 + \mu_B g_s [B_z + B_{\text{nt}}(d)] m_s \} |m_s\rangle \langle m_s| \\ & + \sum_{m_s, m'_s} \frac{1}{2} \mu_B g_s B_0 (e^{i\omega_0 t} + e^{-i\omega_0 t}) \langle m_s | \hat{S}_x | m'_s \rangle |m_s\rangle \langle m'_s| \\ & + \sum_{m_s} \mu_B g_s (\hbar/2m\omega_{\text{nt}})^{1/2} \partial_y B_{\text{nt}} m_s |m_s\rangle \langle m_s| (\hat{a}^\dagger + \hat{a}). \end{aligned} \quad (\text{S11})$$

In the rotating-frame at the driving frequency ω_0 and under the rotating-wave approximation, we can obtain

$$\begin{aligned} \hat{H}_{\text{NV}} = & (\hbar D + \mu_B g_s B_z + B_{\text{nt}} - \hbar\omega_0) | +1\rangle \langle +1| + (\hbar D - \mu_B g_s B_z - B_{\text{nt}} - \hbar\omega_0) | -1\rangle \langle -1| \\ & + \frac{\sqrt{2}}{4} \mu_B g_s B_0 (|0\rangle \langle +1| + | +1\rangle \langle 0|) + \frac{\sqrt{2}}{4} \mu_B g_s B_0 (|0\rangle \langle -1| + | -1\rangle \langle 0|) \\ & + \mu_B g_s (\hbar/2m\omega_{\text{nt}})^{1/2} \partial_y B_{\text{nt}} (| +1\rangle \langle +1| - | -1\rangle \langle -1|) (\hat{a}^\dagger + \hat{a}). \end{aligned} \quad (\text{S12})$$

Including the free Hamiltonian of the vibration mode, we have

$$\begin{aligned} \hat{H}_{\text{NV}} = & \hbar\omega_{\text{nt}} \hat{a}^\dagger \hat{a} + \hbar\Delta_+ | +1\rangle \langle +1| + \hbar\Delta_- | -1\rangle \langle -1| \\ & + \hbar\Omega [| -1\rangle \langle 0| + |0\rangle \langle -1|] + \hbar\Omega [| +1\rangle \langle 0| + |0\rangle \langle +1|] \\ & + \hbar g (| +1\rangle \langle +1| - | -1\rangle \langle -1|) (\hat{a}^\dagger + \hat{a}). \end{aligned} \quad (\text{S13})$$

with $\hbar\Delta_\pm = \hbar D \pm \mu_B g_s (B_z + B_{\text{nt}}) - \hbar\omega_0$, $\hbar\Omega = \frac{\sqrt{2}}{4} \mu_B g_s B_0$, and $\hbar g = \mu_B g_s (\hbar/2m\omega_{\text{nt}})^{1/2} \partial_y B_{\text{nt}}$.

In the following we assume symmetric detunings $\Delta_+ = \Delta_- = \Delta$ for simplicity. We can define the bright and dark states for the NV spin states

$$\begin{aligned} |\mathcal{B}\rangle &= \frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle) \\ |\mathcal{D}\rangle &= \frac{1}{\sqrt{2}}(|+1\rangle - |-1\rangle). \end{aligned} \quad (\text{S14})$$

Then we find that the microwave field couples the state $|0\rangle$ to the bright state $|\mathcal{B}\rangle$, while the dark state $|\mathcal{D}\rangle$ remains decoupled. In the dressed state basis $\{|\mathcal{G}\rangle = \cos\theta|0\rangle - \sin\theta|\mathcal{B}\rangle, |\mathcal{E}\rangle = \cos\theta|\mathcal{B}\rangle + \sin\theta|0\rangle\}$, with $\tan 2\theta = 2\sqrt{2}\Omega/\Delta$, Hamiltonian (S13) can be rewritten as [S5]

$$\begin{aligned} \hat{H}_{\text{NV}} &= \hbar\omega_{\text{nt}}\hat{a}^\dagger\hat{a} + \hbar\omega_{eg}|\mathcal{E}\rangle\langle\mathcal{E}| + \hbar\omega_{dg}|\mathcal{D}\rangle\langle\mathcal{D}| \\ &\quad + \hbar(g_1|\mathcal{G}\rangle\langle\mathcal{D}| + g_2|\mathcal{D}\rangle\langle\mathcal{E}| + \text{H.c.})(\hat{a}^\dagger + \hat{a}), \end{aligned} \quad (\text{S15})$$

where $\omega_{eg} = \sqrt{\Delta^2 + 8\Omega^2}$, $\omega_{dg} = \frac{\Delta + \sqrt{\Delta^2 + 8\Omega^2}}{2}$, $g_1 = -g\sin\theta$, and $g_2 = g\cos\theta$. Under the condition $\Delta \gg \Omega$, one has $\sin\theta \simeq 0$, $\cos\theta \simeq 1$, $\omega_{eg} \simeq \Delta + \frac{4\Omega^2}{\Delta}$, $\omega_{dg} \simeq \Delta + \frac{2\Omega^2}{\Delta}$, and $|\mathcal{E}\rangle \simeq |\mathcal{B}\rangle$, which leads to

$$\hat{\mathcal{H}}_q = \hbar\omega_{\text{nt}}\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\Lambda\hat{\sigma}_z + \hbar g(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a}^\dagger + \hat{a}). \quad (\text{S16})$$

The two-qubit operations

Strong spin-spin interactions mediated by phonons

We consider two NV centers, separated by a distance $l \sim 1\mu\text{m}$, coupled to the same vibration mode of the nanotube, with Hamiltonian

$$\hat{\mathcal{H}}_{2q} = \hbar\omega_{\text{nt}}\hat{a}^\dagger\hat{a} + \sum_{i=1,2} \frac{1}{2}\hbar\Lambda_i\hat{\sigma}_z^i + \sum_{i=1,2} \hbar g_i(\hat{\sigma}_+^i + \hat{\sigma}_-^i)(\hat{a}^\dagger + \hat{a}). \quad (\text{S17})$$

For simplicity, we assume $\Lambda_1 \simeq \Lambda_2 = \Lambda$ and $g_1 \simeq g_2 = g$, and consider the dispersive regime $|\Lambda - \omega_{\text{nt}}| \gg g$, when two NV centers are far detuned from the resonator but in resonance with each other. After the use of a Schrieffer-Wolff transformation, this will lead to an effective nonlocal spin-spin interaction via the exchange of virtual phonons,

$$\hat{\mathcal{H}}_{\text{s-s}} = \hbar\lambda_{\text{eff}}(\hat{\sigma}_+^1\hat{\sigma}_-^2 + \hat{\sigma}_-^1\hat{\sigma}_+^2), \quad (\text{S18})$$

with the coupling strength $\lambda_{\text{eff}} = g^2/|\Lambda - \omega_{\text{nt}}|$. This interaction can extend over distances on the order of the nanotube's length, which allows us to coherently control the interactions between distant NV spins.

We now proceed to discuss the coherence length l_c of the phonon mediated NV spin coupling, which is essential to evaluate the application potential of this device. In particular, it is a very important issue when propagating phonons are considered rather than the confined one used in this work. At ambient temperature, phonon scattering inside solid state materials results in harmful decoherence processes. An important figure of merit that is used to quantitatively characterize all the phonon dissipation mechanisms, including the decay of vibrations into the support as well as intrinsic damping mechanism within the nanotube, e.g. due to scattering from surface defects, is the mechanical quality factor Q , defined as the ratio of the resonant frequency over the linewidth. It is interesting to estimate a sort of coherence length for the nanoresonator mode. To do so one can think of the fundamental bending mode as a traveling wave, which is reflected at the ends of the nanotube. Thus, the coherence length can be estimated as the effective mean phonon free path $l_c \simeq v\tau$ [S6, S7], where v is the effective speed of sound, and τ is the relaxation time (mechanical damping rate $\gamma_m = \tau^{-1}$). The effective speed of sound v can be found from the relation as $\omega_{\text{nt}} = vk_0$ [S8, S9], while the relaxation time is related to the quality factor of the mechanical mode $\frac{1}{\tau} = \omega_{\text{nt}}/Q$. Thus, we can estimate the coherence length of the phonon mediated NV spin coupling as $l_c \sim Q/k_0 \sim QL$. It can extend over distances on the order of several centimeters, much larger than the distance between two NV spins. Therefore, for a high- Q mechanical resonator, phonons can coherently propagate inside it back and forth for quite a long distance before they finally dissipate. This phenomenon has a direct analogy to photons bound in a high- Q micro-cavity. Thus, in this scheme, we can safely ignore the harmful effect of phonon scattering inside solid state materials, provided that the mechanical resonator possesses a very high quality factor at low temperature. We need to emphasize that the recent fabrication of carbon nanotube resonators can possess a quality factor exceeding 10^5 [S10], which ensures that

phonon losses do not severely limit our scheme. The minimum requirement of this work is that the length scale of the phonon mediated NV spin coupling is on the order of the tube's length, which would allow us to coherently control the NV spin interactions, and facilitate potential applications of this hybrid device.

Dynamics of the two coupled NV spins

To implement this protocol, we need a microwave to drive the transition between the qubit state $|0\rangle_j$ and the bright state $|\mathcal{B}\rangle_j$ in each qubit with Rabi frequency Ω_j and frequency detuning δ_j . The dynamics of the entire system is described by

$$\hat{\mathcal{H}} = \sum_j \hbar \delta_j |\mathcal{B}\rangle_{jj} \langle \mathcal{B}| + \sum_j [\hbar \Omega_j |\mathcal{B}\rangle_{jj} \langle 0| + \text{H.c.}] + \hat{\mathcal{H}}_{\text{s-s}}. \quad (\text{S19})$$

The spin-spin interaction can be diagonalized with the states $|\pm\rangle_q = 1/\sqrt{2} [|\mathcal{B}\rangle_1 |\mathcal{D}\rangle_2 \pm |\mathcal{D}\rangle_1 |\mathcal{B}\rangle_2]$, leading to

$$\hat{\mathcal{H}}_{\text{s-s}} = \hbar \lambda_{\text{eff}} |+\rangle_{qq} \langle +| - \hbar \lambda_{\text{eff}} |-\rangle_{qq} \langle -|. \quad (\text{S20})$$

To implement a SWAP gate and quantum information transfer between two qubits, we encode quantum information in the two spin states as $|0\rangle_q = |0\rangle$ and $|1\rangle_q = |\mathcal{D}\rangle$. The entire system is described by

$$\hat{\mathcal{H}} = \sum_j \hbar \delta_j |\mathcal{B}\rangle_{jj} \langle \mathcal{B}| + \sum_j [\hbar \Omega_j |\mathcal{B}\rangle_{jj} \langle 0| + \text{H.c.}] + \hbar \lambda_{\text{eff}} |+\rangle_{qq} \langle +| - \hbar \lambda_{\text{eff}} |-\rangle_{qq} \langle -|. \quad (\text{S21})$$

To gain more insight into the dynamics of the coupled system, we write the Hamiltonian Eq. (S21) in the space S spanned by the state vectors $\{|0, 1\rangle_q, |+\rangle_q, |-\rangle_q, |1, 0\rangle_q, |0, 0\rangle_q, |1, 1\rangle_q, |\mathcal{B}, \mathcal{B}\rangle_q, |0, \mathcal{B}\rangle_q, |\mathcal{B}, 0\rangle_q\}$,

$$\hat{\mathcal{H}} = \hbar \begin{bmatrix} 0 & \bar{\Omega}_1 & \bar{\Omega}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{\Omega}_1 & \delta_+ & 0 & \bar{\Omega}_2 & 0 & 0 & 0 & 0 & 0 \\ \bar{\Omega}_1 & 0 & -\delta_- & -\bar{\Omega}_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{\Omega}_2 & -\bar{\Omega}_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_2 & \Omega_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_1 + \delta_2 & \Omega_1 & \Omega_2 \\ 0 & 0 & 0 & 0 & \Omega_2 & 0 & \Omega_1 & \delta_2 & 0 \\ 0 & 0 & 0 & 0 & \Omega_1 & 0 & \Omega_2 & 0 & \delta_1 \end{bmatrix} \quad (\text{S22})$$

From the matrix form for the Hamiltonian Eq. (S21), we see that the space S can be decomposed into two independent subspaces $S_1 = \{|0, 1\rangle_q, |+\rangle_q, |-\rangle_q, |1, 0\rangle_q\}$ and $S_2 = \{|0, 0\rangle_q, |1, 1\rangle_q, |\mathcal{B}, \mathcal{B}\rangle_q, |0, \mathcal{B}\rangle_q, |\mathcal{B}, 0\rangle_q\}$, i.e., $S = S_1 \oplus S_2$. Thus we can find that if the two qubits are initially prepared in the state $|0\rangle_q^1 |1\rangle_q^2$ or $|1\rangle_q^1 |0\rangle_q^2$, then the dynamics of the system will be confined in the subspace S_1 governed by the Hamiltonian

$$\begin{aligned} \hat{\mathcal{H}} = & \hbar \delta_+ |+\rangle_{qq} \langle +| - \hbar \delta_- |-\rangle_{qq} \langle -| + \hbar \bar{\Omega}_1 |+\rangle_{qq} \langle 0, 1| \\ & + \hbar \bar{\Omega}_1 |-\rangle_{qq} \langle 0, 1| + \hbar \bar{\Omega}_2 |+\rangle_{qq} \langle 1, 0| - \hbar \bar{\Omega}_2 |-\rangle_{qq} \langle 1, 0| + \text{H.c.} \end{aligned} \quad (\text{S23})$$

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